Rules for Differentiation

Derivative of a Constant Function

If f is the function with the constant value c, then

$$\frac{df}{dx} = \frac{d}{dx}\left(c\right) = 0$$

"Proof"

-If $f\left(x\right) = c$ is a function with a constant value c, then

$$\lim_{h \to 0} \frac{f\left(x+h\right) - f\left(x\right)}{h} = \lim_{h \to 0} \frac{c-c}{h} = \frac{0}{h} = 0$$

Power Rule for Positive Integer Powers of x

If n is a positive integer, then

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Constant Multiple Rule

If u is a differentiable function of x and c is a constant, then

$$\frac{d}{dx}\left(cu\right) = c\frac{du}{dx}$$

Sum and Difference Rule

If u and v are differentiable functions of x, then their sum and difference are differentiable at every point where u and v are differentiable.

$$\frac{d}{dx}\left(u \pm v\right) = \frac{du}{dx} + \frac{dv}{dx}$$

Example

Find $\frac{dp}{dt}$ if $p = t^3 + 6t^2 - \frac{5}{3}t + 16$

$$\frac{dp}{dt} = \frac{d}{dt}\left(t^3\right) + \frac{d}{dt}\left(6t^2\right) - \frac{d}{dt}\left(\frac{5}{3}t\right) + \frac{d}{dt}\left(16\right)$$

$$= 3t^2 + 12t - \frac{5}{3} + 0$$

$$= 3t^2 + 12t - \frac{5}{3}$$

Example

Does $y = x^4 - 2x^2 + 2$ have any horizontal tangents?

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^4 - 2x^2 + 2\right) = 4x^3 - 4x$$

$$4x^3 - 4x = 0$$

$$4x\left(x^2 - 1\right) = 0$$

$x = 0, 1, -1$ 'Graph it!

The Product Rule

-Unlike sum and difference the derivative of the product of two functions is NOT the product of the derivatives.

$$\frac{d}{dx}\left(x \bullet x\right) = \frac{d}{dx}\left(x^2\right) = 2x$$

'NOT the same!!

$$\left[\frac{d}{dx}\left(x\right)\right]\left[\frac{d}{dx}\left(x\right)\right] = 1 \bullet 1 = 1$$

-The product of two differentiable functions u and v is differentiable and

$$\frac{d}{dx}\left(uv\right) = u\frac{dv}{dx} + v\frac{du}{dx}$$

"The first times the derivative of the second plus the second times the derivative of the first."

$$\text{“}f \circ g' \quad g \circ f'\text{”}$$

Example

Find $f'\left(x\right)$ if $f\left(x\right) = \left(x^2 + 1\right)\left(x^3 + 3\right)$

$$f'\left(x\right) = \frac{d}{dx}\left[\left(x^2 + 1\right)\left(x^3 + 3\right)\right]$$

$$= \left(x^2 + 1\right)\left(3x^2\right) + \left(x^3 + 3\right)\left(2x\right)$$

$$= 3x^4 + 3x^2 + 2x^4 + 6x$$

$$= 5x^4 + 3x^2 + 6x$$

Example-Numerical Values

Let $y = uv$ be the product of the functions u and v

Find $y'(2)$ if

$$u(2) = 3,\ u'(2) = -4,\ v(2) = 1,\ v'(2) = 2$$

$y' = (uv)' = uv' + vu'$

$y'(2) = u(2)v'(2) + v(2)u'(2)$

$= 3(2) + (1)(-4)$

$= 2$

Quotient Rule

At a point where $v \neq 0$, the quotient $y = \frac{u}{v}$ of two different functions is differentiable, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{g \circ f' \ f \circ g'}{g^2}$$

Example

Find y' if $y = \dfrac{5x-2}{x^2+1}$

$$y' = \frac{(x^2+1)(5)-(5x-2)(2x)}{(x^2+1)^2}$$

$$= \frac{5x^2+5-10x^2+4x}{(x^2+1)^2}$$

$$= \frac{-5x^2+4x+5}{(x^2+1)^2}$$